CHAPTER 10
THE DIVISIBILITY OF MOTION

No continuum is composed of indivisibles (Book 6, Lesson 1)

If continuous things are those whose extremities are one (e.g. a line), it is impossible for it to be composed of indivisibles (e.g. points in a line). If something is composed of parts, the extremities must either be one (**continuous**) or they must be together (**contiguous**). But the extremities of points cannot be one or together, because an extremity is spoken of in relation to a part, whereas an indivisible is not related to any part. Besides, if a continuum is composed solely of points, they cannot touch, since everything that touches another does so by a part touching the other. But since an indivisible has no parts, a whole point must touch a whole point without distinction as to place or position; such a conjunction cannot result in a continuum.

Nor can a continuum be composed of **consecutive** indivisibles. This is because consecutive things, by definition, have nothing of the same kind intervening. But between any two points there is always a line, and if a line is composed of points only, then between any two points there is always another mediate point. The same is true for the "nows" of time; no time can be composed solely of "nows". Besides, a continuum is "that which is divisible *ad infinitum*", but if a continuous line is composed of indivisibles, then these indivisibles would have to be divisible, which is nonsense.

Thus between two points there must always be a line, and between two "nows" there must always be time. That is because, if two points exist, they must differ in position; otherwise they would not be two, but one. No other intermediate is possible except a line between two points and time between two "nows", because if the intermediate ever divisible into indivisibles the same problem would return as above–how a divisible can be composed solely of indivisibles. If the intermediate is always divisible into further divisibles, then it is a continuum.

No motion is composed of indivisibles (Book 6, Lesson 2)

Magnitude and motion are correlative, so that if magnitude is composed of indivisibles, so must motion, which traverses it. And if this is impossible for magnitude, then it is also impossible for motion.

To illustrate this, take a magnitude (or road) that consists of points A, B and C. When the mobile reaches B point it must either be in motion or have completed its motion. If it is still in motion, then B must be divisible; otherwise the motion would be complete at that point and the sum of the motion from A to C would not be divisible motions but discrete moments [like frames in a video].

Then, if motion consists of discrete moments, it would follow that something has completed a motion without having been in motion. Also it would follow that at each point on the route the mobile would be at rest, while it was supposed to be in motion. Also the segments of motion corresponding to each of the points on the route would also be rests, and thus the whole motion would be composed of non-motions.

No time is composed of indivisibles (Book 6, Lesson 3)

Time is divisible just as the magnitude being traversed in time. Thus a mobile going at an equal speed covers half the distance in half the time of its journey. Therefore the two are correlatively divisible into smaller and smaller segments.

The same can be seen from mobiles of different velocity: When the fastest reaches the finish line, the other is at some intermediate point; likewise the faster mobile reached that intermediate point in less time. This shows that the magnitude and the time are equally divisible. As we increase velocity, we divide time, since the journey is finished in shorter and shorter times. But if we decrease the velocity

and stop the mobile after the same length of time, we divide the magnitude, since less and less distance is traversed.

Although there are physical limits to division of a magnitude, just as there are physical limits to the size of a natural thing, so there are natural limits to velocity, but mathematically both magnitude and time are infinitely divisible, and the same magnitude is traversable in ever faster speeds.

No continuum is indivisible (Book 6, Lesson 4)

Because time corresponds to magnitude, if magnitude could be infinite, such as a line without beginning or end point, so would time be (since any finite time at a finite velocity will only traverse a finite magnitude). And if magnitude is infinitely divisible, so is time. The argument can validly be reversed: If time is infinite in length, so must magnitude be (because any motion, however slow will eventually traverse a finite magnitude); and if time is infinitely divisible, so must magnitude be.

From this we have another argument why no continuous magnitude or part thereof is indivisible. This can be shown inductively by positing two mobiles of different velocities. When the faster one crosses the first supposedly indivisible part, the slower one will have crossed only part way, proving that the supposedly indivisible segment is divisible.

The indivisibility of "now" and divisibility of the motion (Book 6, Lesson 5)

The term "now" is often used for a period, like "today", but "now" is precisely indivisible and present in every time. It is the limit between the past and the future, and as such must be one. The now terminating the past and beginning the future cannot be two contiguous "nows", because otherwise time would be composed of an aggregate of indivisible "nows", which is impossible, since it is continuous. Nor can there be an interval between the two "nows", because that would be a period of time or at least another "now"; in either case the "now" would be divisible. Were the "now" to be divisible it would have to include some of the past or the future; in that case some of the future would be in the past and some of the past in the future. Therefore the "now" which terminates the past and begins the future must be one "now".

Consequently, there can be no motion in a "now", since motion is continuous, while the "now is not; this can be proved (as above) by positing two mobiles going at different velocities; if there were motion in the "now", the faster would have traversed the same distance in less than a "now", making it divisible.

Likewise there can be no rest in a "now"; that is because rest is the privation of motion, and there is no privation where there is no aptitude to have the object of privation. So, if there can be no motion in the "now", there can be no rest either. Also, should the "now" be the termination of motion and the beginning of rest, then, were there to be rest in the "now", something would be both in motion and at rest in the same now. Rather, rest is understood as something continuous, like motion.

All motion occurs in time and must be intermediate between two extremes, with part of the mobile towards one extreme and part of it towards the other. This is obvious of the three species of motion, but, even though generation and corruption are substantial changes and instantaneous, they are preceded by alteration of a subject. Some alteration, like electricity in a wire, seems instantaneous, but precise measurements can show that it too takes place in time.

Two ways motions is divided (Book 6, Lesson 6)

Motion is divided according to the motion of the parts of the mobile, since both time and the mobile is continuous and divisible. Thus the entire motion belongs to the entire mobile, just as the parts of it belong to the parts of the mobile. Motion is also divided according to time, since there is less motion in less time.

In fact, there are five things related to motion which are similarly divided: (1) time, (2) motion, (3) the very act of being moved, (4) the mobile which is being moved, and (5) the species of motion, i.e. the place, quality or quantity. The divisibility of the mobile is the basis of the divisibility of all the others.

The beginning and end of motion are indivisibles (Book 6, Lesson 7)

It is first obvious that substantial change is the instantaneous moment when something ceases to be what it was and becomes something else. But the end of any motion is similarly and indivisible moment, when something can first be said to "have changed".

The beginning of a motion is likewise an indivisible moment. It is impossible to assign a first time when a thing began moving, because any time is divisible.

Every "being moved" is preceded by a "having been moved" and every "having been moved by a "being moved" (Book 6, Lesson 6)

Since motion is continuous and infinitely divisible, while anything is being moved, any past point of the motion can be designated where the mobile can be said to "have been moved", and in which there is no motion. This point marks a part of the motion which has been completed. Such a point is only potentially a term of motion, since the motion did not stop there.

Any such point that is selected as marking "the journey completed thus far" is preceded by another segment of the motion, or a "being moved"; otherwise it would be the beginning point of the motion.

These two statements can be proved, as above, by showing how two mobiles of different speeds mark segments of motion. They do not apply directly to generation and corruption, since these are not continuous motions but an instantaneous change. Nevertheless, generation and corruption are the term of a process of alteration, and the alteration, named after its term, is divisible: Thus "dying" is a motion that can be divided into stages.

Magnitude, motion, time and the mobile are all infinite or finite in the same way (Book 6, Lesson 9)

If magnitude (i.e. the path) is finite, time cannot be infinite, and if time is finite magnitude cannot be infinite; this can be shown by a comparative multiplying of the finite or dividing the infinite.

By similar arguments it can be shown that a mobile cannot be infinite if either the magnitude or the time is finite. The same thing can be said about motion.

The division of rest (Book 6, Lesson 10)

First, it is clear that "coming to rest" is part of motion, and this takes place in time. Just as we have seen that no part of motion can be said to be **first**, so no part of coming to rest can be said to be first.

Likewise, rest itself, being continuous in time, can have no first part, since each part is divisible. Something is at rest if throughout a definite period of time (from one "now" to another) it is one and same state, for example in one place. Therefore nothing can be at rest and at motion at the same time with respect to the same kind of motion.

Refutation of Zeno's denial of the possibility of motion (Book 6, Lesson 11)

The following are some of Zeno's reasons to show that local motion is impossible:

1. To traverse any space, a mobile must first go half way; since any space is infinitely divisible and the infinite cannot be traversed in finite time, nothing can be moved. The answer to this is that the infinite points in any distance are not actual but only potential.
2. Similarly, a faster mobile can never catch up with a slower mobile that started earlier, because

likewise it must first reach the point where the first mobile was. As it gains on the first mobile the points become nearer and nearer by infinite division, so that it never catches up. The answer to this is the same as above, since these points are not actual but only potential.

3. Time is made up of instants, and in every instant of time there is rest and not motion. The answer to this is that time is composed of only potential, not actual instants. Besides, just as there is no motion in an instant, so there is no rest.

Indivisibles are incapable of *per se* motion (Book 6, Lesson 12)

Democritus thought that indivisible atoms are *per se* mobile. But points exist only as the terminations of lines and move accidentally as the body in which they are found is moved. But granted that indivisible atoms could exist on their own, they could not be partly in one place and partly in another as extended mobiles are while in motion. Therefore they would have to cross the distance point by point and "now" by "now", and that is impossible. Therefore they would have to be permanently at rest.

No change is infinite (Book 6, Lesson 13)

Changes which are between contradictory terminals, which are affirmation or negation of something, as in the case of generation and corruption, are instantaneous and do not go on for any time, much less forever.

Changes that are between contrary terminals have a maximum and minimum term according to the nature of the subject and the species of change: thus every alteration has some limit. The same can be said for growth and decrease, because each nature has a size that befits it: there is one for a man and another for a horse. Therefore none of these changes can be infinite.

The same cannot be said of local motion, because not every local motion is between strict contraries, where contraries refer to things most distant. Gravitational motion has a definite term, that is the place where the falling body rests; so this is not infinite. But for motion to endure throughout infinite time in such a way that it remains one numerical motion can occur only in circular local motion; thus orbital motion can endure as one and continuous throughout infinite time, as will be seen later.